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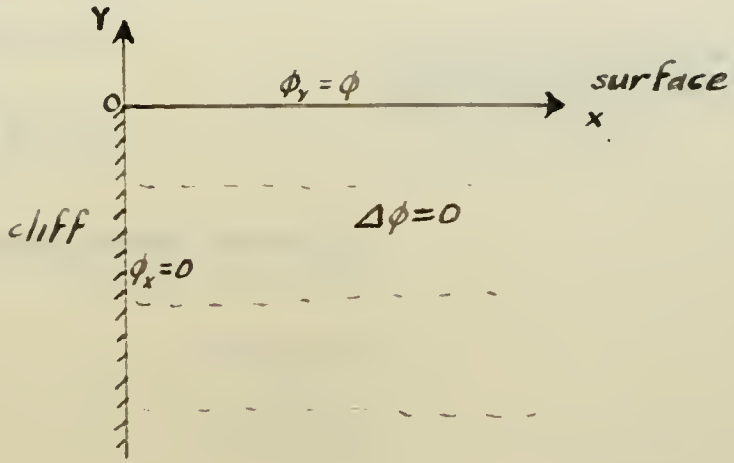
# WATER PARTICLE PATHS NEAR A VERTICAL BARRIER

At the suggestion of Dr. Hans Lewy of the University of California, Berkeley, California (formerly with AMG-NYU), we have carried out the calculations necessary to determine the water particle paths near a cliff against which waves are moving. On the basis of these results a general curve is drawn, along which the tangent has the direction of the major axis of the elliptical particle paths. It is proposed that constructing such a rounded bottom near the wall of an experimental tank will help to cancel some unwanted bottom effects.

The general theory describing a method of obtaining progressing waves is expounded in an AMG-NYU report soon to appear. However, in the case of a 90° cliff,




we simply write down two potential functions $\phi_1(x,y)$ and $\phi_2(x,y)$ which satisfy the appropriate boundary conditions and are 90° "out of phase" on the surface $y = 0$, for $x \to \infty$:

$$1)\quad \begin{cases} \phi_1 = \mathrm{Re}\ \pi e^{-iz} = \pi e^{y} \cos x & (z = x + iy;\ \mathrm{Re} = \text{Real part of}) \\ \phi_2 = \mathrm{Re}\left[ e^{-iz} \int_{\infty}^{-iz} \frac{e^{-u}}{u}\, du + \pi i e^{-iz} \right]^{*} \cong \pi e^{y} \sin x & (\text{for large } x) \end{cases}$$

To obtain a progressing wave, we set

$$2)\quad \Phi(x,y;t) = \phi_1(x,y) \sin t + \phi_2(x,y) \cos t$$

(t represents dimensionless time, $\sigma\bar{t}$)

Therefore, for large x

$$\Phi(x,y;t) \cong \pi e^{y}[\cos x \sin t + \sin x \cos t] = \pi e^{y} \sin (x + t).$$

The particles of water, $[x(t), y(t)]$, satisfy the following system of differential equations:

$$3)\quad \begin{cases} \dfrac{dx}{dt} = -\Phi_x(x,y;t) \\ \dfrac{dy}{dt} = -\Phi_y(x,y;t) \end{cases}$$

(the subscript meaning partial differentiation with respect to that variable)

together with some initial conditions,

$$x(t_o) = x_o$$

$$y(t_o) = y_o.$$

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* We are employing the appropriate dimensionless lengths x and y where, if we set $\lambda$ = wave length at $\infty$, $\frac{\lambda}{2\pi}x$ and $\frac{\lambda}{2\pi}y$ represent true distances (i.e. $\lambda = \frac{2\pi g}{\sigma^2}$, where g = acceleration due to gravity, $\sigma$ = frequency of standing waves, $\phi e^{i\sigma\bar{t}}$).

We have integrated* the equations of motion 3) for several initial points and have verified that the following simple description of the particle paths is correct: Any individual particle path is fundamentally elliptical in shape--the "ellipses" are large near the surface because the velocities are large and decrease in size as we go deeper. On the other hand the principal axis (say) of the ellipse has a slope which is constant on lines (curves) which are almost parallel to the y-axis (i.e., x = constant). The slopes are periodic functions in x (on a line y = constant). The accompanying plot of the vector field $(\frac{dx}{dt}, \frac{dy}{dt})$ at two times which are $90^{\circ}$ "out of phase" indicates the periodicity property. The blue, broken line curves, are finite difference solutions to the equations of motion 3) for three initial points in the plane. (Two different intervals in time were used in one case and the resulting two blue curves indicate that the curve resulting from the use of the finer time interval is sufficiently accurate.)

We have roughly drawn a curve which has the property that its tangent coincides with the slope of the principal axis of the "ellipse" about any point on the curve. This master curve is roughly independent of the depth of its minimum point and can be shifted vertically upward or downward parallel to itself.

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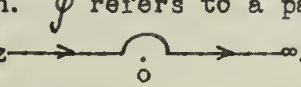
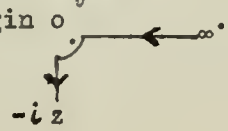
* A short table of the exponential integral

$$E(z) = e^{z} \oint_{z}^{\infty} \frac{e^{-u}}{u}\, du \qquad \text{(for complex values of } z\text{)},$$

prepared by the Mathematical Tables Project, has proved of value in the computation. $\oint$ refers to a path of integration which stays above the origin z  $\infty$. $\oint$ refers to a path of integration which stays below the origin o  $\infty$.

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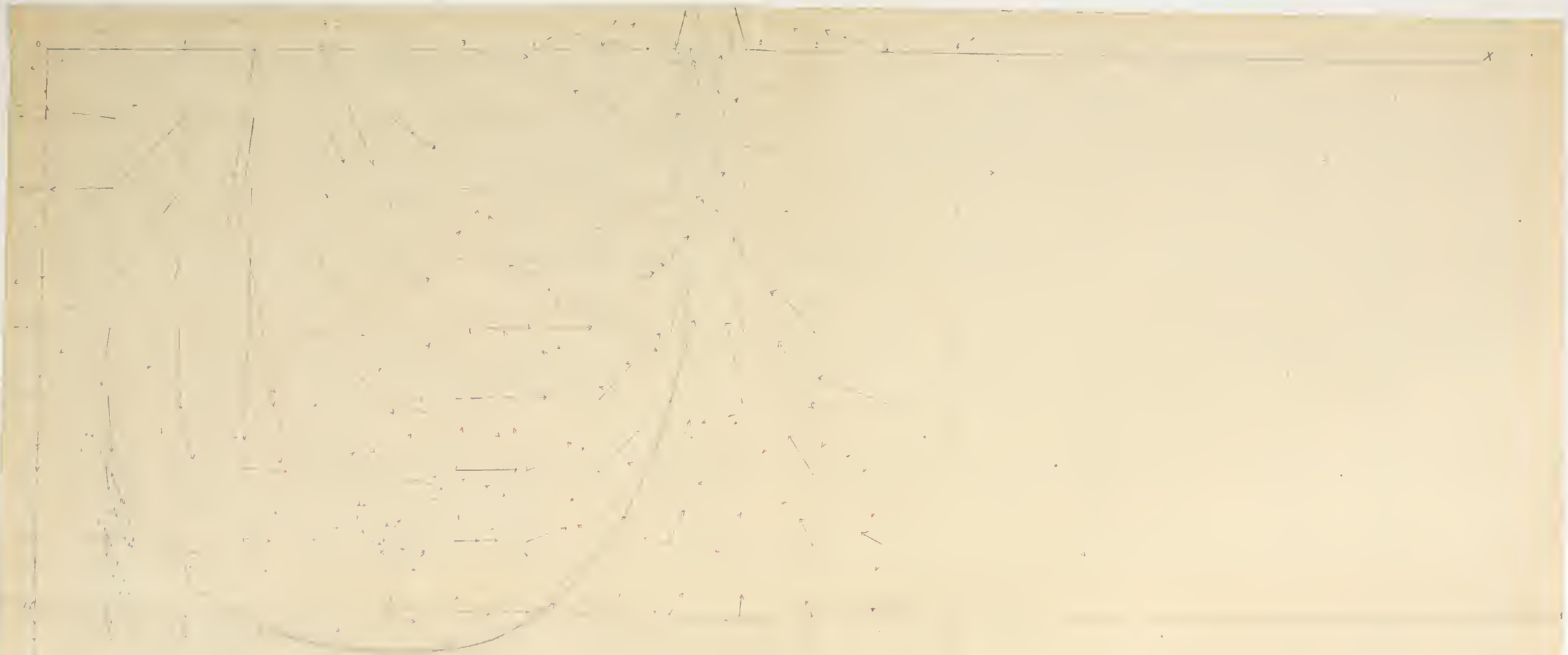

$x = \frac{2\pi \bar{x}}{\lambda}$ , $\lambda$ = wavelength at $\infty$ $= \frac{2\pi g}{\sigma^2}$

$\bar{x}$ = distance from dock

$y = \frac{2\pi \bar{y}}{\lambda}$ , $\bar{y}$ = distance from surface

Black vectors are velocity vectors for non-homogeneous solution $[\varphi_2(x,y)]$ (i.e. $\varphi_2(x,y)$ is singular at $(0,0)$)

Red vectors are velocity vectors for homogeneous solution $[\varphi_1(x,y)]$ (i.e. $\varphi_1(x,y)$ is regular everywhere)

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